3.3.2 Plane-Strain Problems

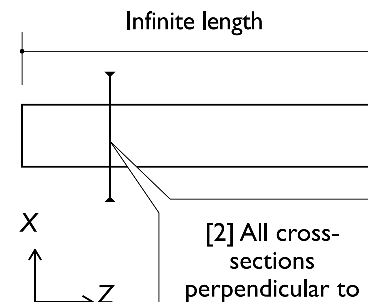
Plane-Strain Condition

[1] Consider a structure of INFINITE LENGTH in Z-direction. The Z-direction is restrained such that no particles can move in Z-direction. Further, all cross-sections perpendicular to the Z-direction have the same geometry, supports, and loads [2]. In such a case, the strain state at any point can be depicted in [3]. Note that there are no strains in the Z-face; i.e., $\varepsilon_Z = 0$ (otherwise the particle on the Z-face would move in Z-direction) and $\gamma_{ZX} = \gamma_{ZY} = 0$ (otherwise the cube would twist in ZX and ZY planes respectively, and that implies the particles would move in Z-direction),

$$\varepsilon_Z = 0, \quad \gamma_{ZX} = 0, \quad \gamma_{ZY} = 0 \qquad (1)$$

Eq. (1) is called a *plane-strain condition*. If the plane-strain condition holds everywhere, then it is called a plane-strain problem.

In the real world, there is no such thing as infinite length. In practice, a problem may assume the plane-strain condition if its Z-direction is restrained from expansion or contraction and all cross-sections perpendicular to the Z-direction have the same geometry, supports, and loads. As an example, a pressurized pipe buried under the earth is often considered as a plane-strain problem. Section 14.3 and the exercise problems in 3.6.2 (page 171) provide two examples for plane-strain problems.

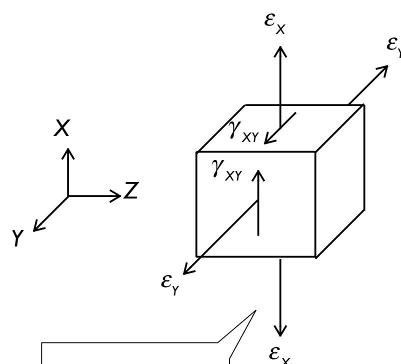


[2] All cross-sections perpendicular to the Z-direction have the same geometry, supports, and loads. ↓

Hooke's Law for Plane-Strain Problems

Eq. 1.2.8(1) (page 31), Hooke's law, can be inverted and rewritten as

$$\begin{aligned}
\sigma_X &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_X + \nu\varepsilon_Y + \nu\varepsilon_Z\right] \\
\sigma_Y &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_Y + \nu\varepsilon_Z + \nu\varepsilon_X\right] \\
\sigma_Z &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_Z + \nu\varepsilon_X + \nu\varepsilon_Y\right] \\
\tau_{XY} &= G\gamma_{XY}, \quad \tau_{YZ} = G\gamma_{YZ}, \quad \tau_{ZX} = G\gamma_{ZX}
\end{aligned} \qquad (2)$$



[3] The strain state at a point in plane strain condition. ↵

The proof of Eq. (2) is in 3.3.14 (page 150).

Substitute the plane-strain condition Eq. (1) into Eq. (2), and Hooke's law becomes

$$\begin{aligned}
\sigma_X &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_X + \nu\varepsilon_Y\right] \\
\sigma_Y &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_Y + \nu\varepsilon_X\right] \\
\sigma_Z &= \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_X + \nu\varepsilon_Y\right] \\
\tau_{XY} &= G\gamma_{XY}, \quad \tau_{YZ} = 0, \quad \tau_{ZX} = 0
\end{aligned} \qquad (3) \nearrow$$