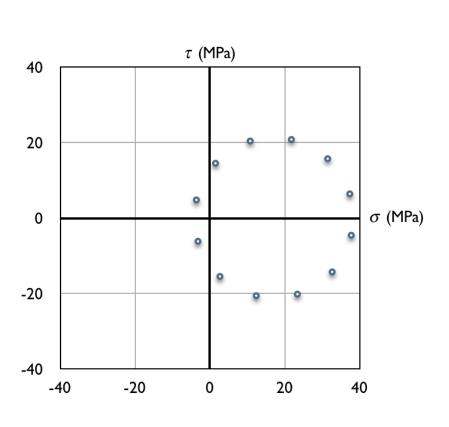
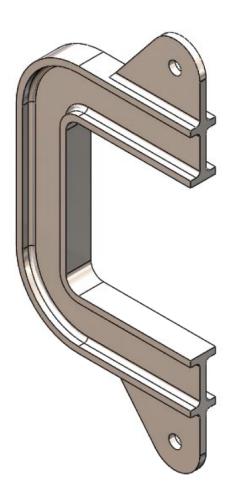
of the exercise of the book

# Mechanics of Materials Labs

with SOLIDWORKS Simulation 2015





Huei-Huang Lee



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# Chapter I

## Stresses

Stresses are quantities used to describe the intensity of force inside a body (either solid or fluid). Its unit is force per unit area (i.e.,  $N/m^2$  in SI). It is a position-dependent quantity.

Imagine that your two arms are pulled by your friends with two forces of the same magnitude, but opposite directions. What are the stresses in your arms? Assuming the magnitude of the forces is 100 N and the cross-sectional area of your arms is  $100 \text{ cm}^2$ , then you may answer, "the stresses are 1 N/cm² everywhere in my arms." This case is simple; the answer is good enough. For a one-dimensional case like this, the stress  $\sigma$  may be easily defined as

$$\sigma = \frac{P}{A}$$

where *P* is the applied force and *A* is the cross sectional area.

In 3D cases, things are much more complicated. Now, imagine that you are buried in the soil by your friends, and your head is 100 meters deep below the ground surface. How do you describe the force intensity (i.e., stress) on your head?

If the soil is replaced by still water, then the answer would be much simpler. The magnitude of the pressure (stress) on the top of your head would be the same as the pressure on your cheeks, and the direction of the pressure would always be perpendicular to the surface on which the pressure applies. You've learned these in your high school. And you've learned that the magnitude of the pressure is  $\sigma = \rho g h$ , where  $\rho$  is the mass density of the water (1000 kg/m³), g is the gravitational acceleration (9.81 m/s²), and h is the depth (100 meters in this case). In general, to describe the force intensity at a certain position in still water, we place an infinitesimally small body at that position, and measure the force per unit surface area on that body.

In the soil (which is a solid material rather than water), the behavior is quite different. First, the magnitude of the pressure on the top of your head may not be the same as that on your cheeks. Second, the direction of pressure is not necessarily perpendicular to the surface on which the pressure applies. However, the above definition of stresses for water still holds. Let me restate as follows:

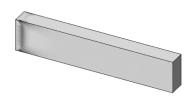
The stress at a certain position in a solid material is defined as the force per unit surface area on an infinitesimally small body placed at that position.

Note that the infinitesimally small body could be any shapes. However, if we know the stresses on a certain shape of a small body, we can infer the stresses on other shapes. We usually take a small cube to describe the stresses.

This chapter will guide you to learn the concepts of stresses.

# Section I.I

#### **Stress Components**



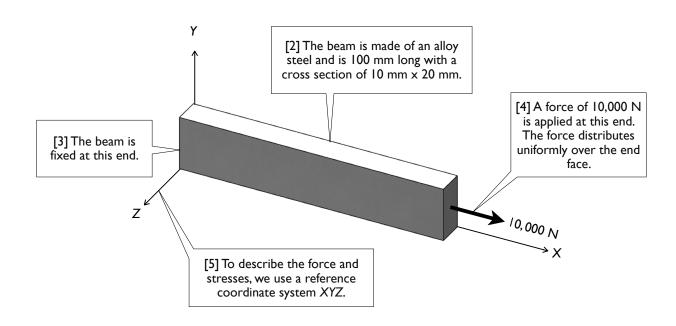
#### I.I-I Introduction

[1] Consider a cantilever beam made of an alloy steel and with dimensions of 10 mm x 20 mm x 100 mm [2], which is fixed at one end [3] and subjected to a force on the other end [4]. The force is in positive X-direction and has a magnitude of 10,000 N. Note that we've used a reference coordinate system as shown in [5].

In this simple case, the stress is uniform over the body; i.e., every point in the beam has the same stress. How do we describe this stress? Can we simply say, the stress is 50 MPa, which is calculated by

$$\frac{10,000 \text{ N}}{10 \text{ mm} \times 20 \text{ mm}} = 50 \text{ MPa}$$
?

For a simple case like this, that may be adequate. In order to apply to more general cases, we need to say something more, specifically, what is the direction of the stress? What is the surface on which the stress acts?



#### **Definition of Stress**

[6] The stress at a certain point can be defined as the force per unit area acting on the boundary surfaces of an infinitesimally small body centered at that point [7]. The stress values may be different at different faces and the small body can be any shape. However, for the purpose of describing the stress, we usually use an infinitesimally small cube [8] of which each edge is parallel to a coordinate axis. If we can find the stresses on a small cube, we then can calculate the stresses on any other shapes of small bodies [18].

#### X-Face, Y-Face, and Z-Face

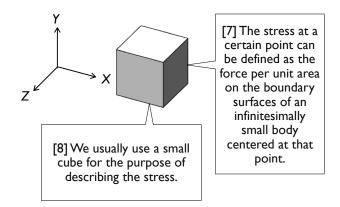
[9] Each of the six faces of the cube can be assigned an identifier, namely X-face, Y-face, Z-face, negative-X-face, negative-Y-face, and negative-Z-face, respectively [10-13]. Note that X-face has X-axis as its outer normal vector, and so on.

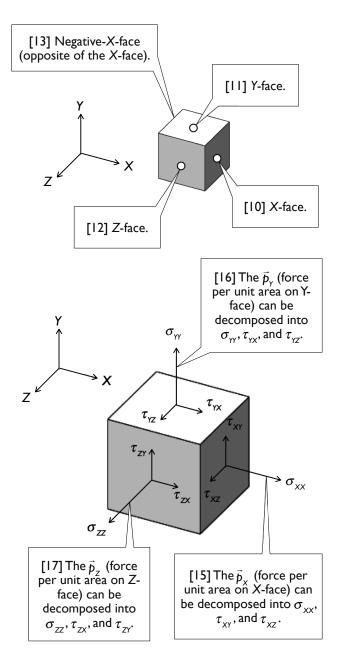
#### **Stress Components**

[14] Let  $\bar{p}_{\chi}$  be the force per unit area acting on the X-face. In general,  $\bar{p}_{\chi}$  may not be normal or parallel to the X-face. We may decompose  $\bar{p}_{\chi}$  into X-, Y-, and Z-component, and denote  $\sigma_{\chi\chi}$ ,  $\tau_{\chi\gamma}$ , and  $\tau_{\chi\chi}$ , respectively [15]. The first subscript (X) is used to indicate the **face** on which the stress components act, while the second subscript (X,Y, or Z) is used to indicate the **direction** of the stress components. Note that  $\sigma_{\chi\chi}$  is normal to the face, while  $\tau_{\chi\gamma}$ , and  $\tau_{\chi Z}$  are parallel to the face. Therefore,  $\sigma_{\chi\chi}$  is called a **normal stress**, while  $\tau_{\chi\gamma}$ , and  $\tau_{\chi Z}$  are called **shear stresses**. In Mechanics of Materials, we usually use the symbol  $\sigma$  for a normal stress and  $\tau$  for a shear stress.

Similarly, let  $\vec{p}_{\gamma}$  be the force per unit area acting on the Y-face and we may decompose  $\vec{p}_{\gamma}$  into a normal component  $(\sigma_{\gamma\gamma})$  and two shear components  $(\tau_{\gamma\chi})$  and  $\tau_{\gamma Z}$  [16]. Also, let  $\vec{p}_{Z}$  be the force per unit area acting on the Z-face and we may decompose  $\vec{p}_{Z}$  into a normal component  $(\sigma_{ZZ})$  and two shear components  $(\tau_{ZX})$  and  $(\tau_{ZY})$  [17]. Organized in a matrix form, these stress components may be written as

$$\left\{ \sigma \right\} = \left( \begin{array}{ccc} \sigma_{XX} & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_{YY} & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_{ZZ} \end{array} \right) 
 \tag{I}$$





#### Stress Components on Other Faces

[18] It can be proven that the stress components on the negative-X-face, negative-Y-face, and negative-Z-face can be derived from the 9 stress components in Eq. (1). For example, on the negative-X-face, the stress components have exactly the same stress values as those on the X-face, but with opposite directions [19]. Similarly, the stress components on the negative-Y-face have the same stress values as those on the Y-face, but with opposite directions [20], and the stress components on the negative-Z-face have the same stress values as those on the Z-face, but with opposite directions [21].

The proof can be done by taking the cube as a free body and applying the force equilibria in X, Y, and Z directions, respectively.

On an arbitrary face (which may not be parallel or perpendicular to an axis), the stress components also can be calculated from the 9 stress components in Eq. (1). We'll show that this can be done using Mohr's circles (Section 10.1).

#### Symmetry of Shear Stresses

[22] It also can be proven that the shear stresses are symmetric, i.e.,

$$\tau_{xy} = \tau_{yx}, \quad \tau_{yz} = \tau_{zy}, \quad \tau_{zx} = \tau_{xz}$$
(2)

The proof can be done by taking the cube as a free body and applying moment equilibria in X, Y, and Z directions, respectively.

#### Stress State

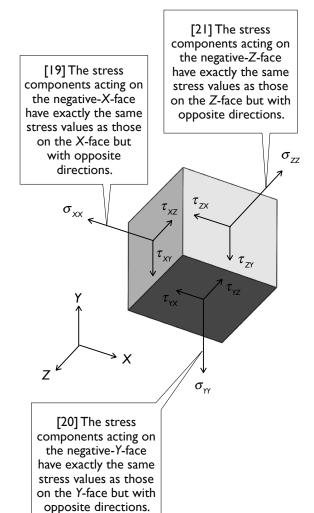
[23] We now conclude that 3 normal stress components and 3 shear stress components are needed to describe the **stress state** at a certain point, which may be written in a vector form

$$\left\{\sigma\right\} = \left\{ \begin{array}{cccc} \sigma_{X} & \sigma_{Y} & \sigma_{Z} & \tau_{XY} & \tau_{YZ} & \tau_{ZX} \end{array} \right\} \tag{3}$$

Note that, to be more concise, we use  $\sigma_{\rm X}$  in place of  $\sigma_{\rm XX}$ ,  $\sigma_{\rm Y}$  in place of  $\sigma_{\rm YX}$ , and  $\sigma_{\rm Z}$  in place of  $\sigma_{\rm ZZ}$ .

The purpose of this section is to familiarize students with the 6 stress components in Eq. (3). The stress field in this section is uniform over the entire body. In the next section, we'll explore a nonuniform stress field.

Another purpose of this section is to familiarize the students with the **SOLIDWORKS Simulation** user interface. #



#### 1.1-2 Launch **SOLIDWORKS** and Create New Part



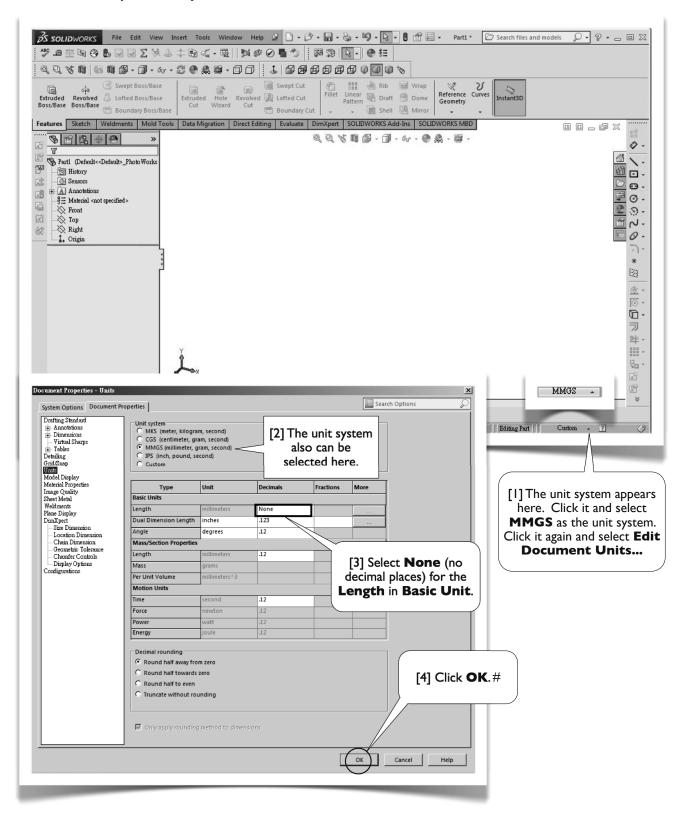
#### About the Textboxes

- I. Within each subsection (e.g., I.I-2), textboxes are ordered with numbers, each of which is enclosed by a pair of square brackets (e.g., [1]). When you read the contents of a subsection, please follow the order of the textboxes.
- 2. The textbox numbers are also used as reference numbers. Inside a subsection, we simply refer to a textbox by its number (e.g., [1]). From other subsections, we refer to a textbox by its subsection identifier and the textbox number (e.g., 1.1-2[1]).
- 3. A textbox is either round-cornered (e.g., [1, 3, 5]) or sharp-cornered (e.g., [2, 4]). A round-cornered textbox indicates that **mouse or keyboard actions** are needed in that step. A sharp-cornered textbox is used for commentary only; i.e., mouse or keyboard actions are not needed in that step.
- 4. A symbol # is used to indicate the last textbox of a subsection [5], so that you don't leave out any textboxes.

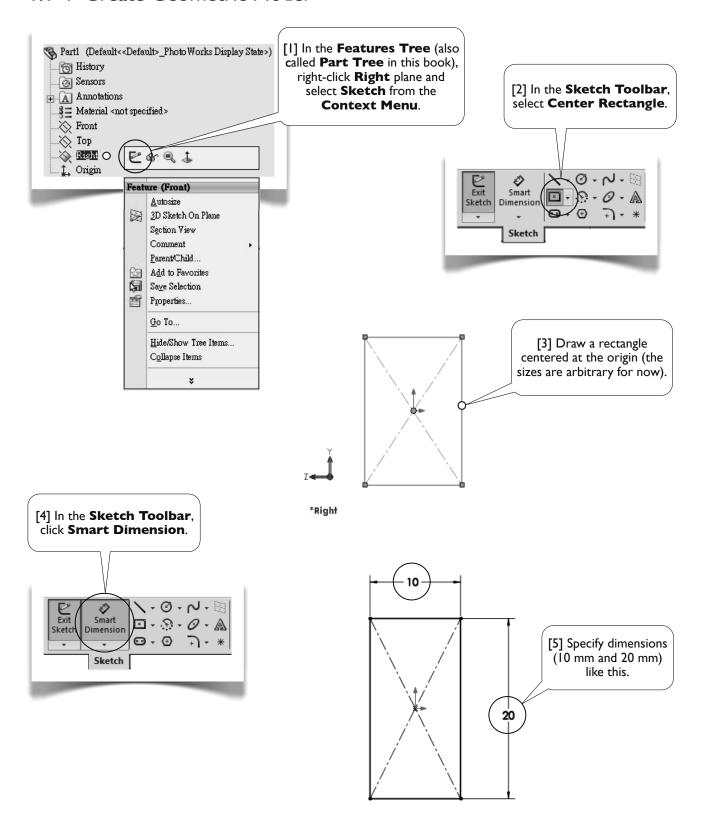
#### **SOLIDWORKS** Terms

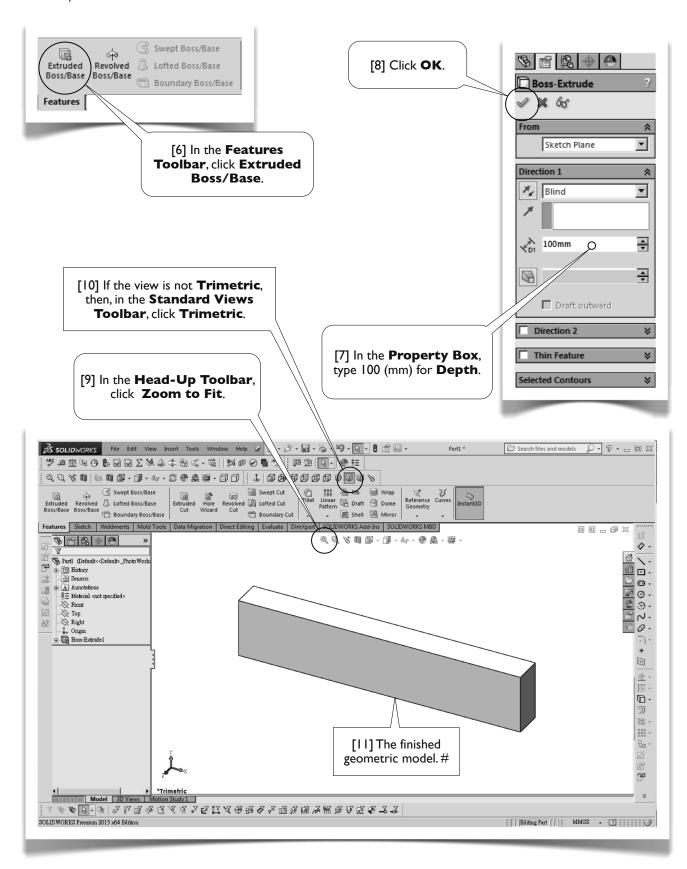
In this book, terms used in the **SOLIDWORKS** are boldfaced (e.g., **Part** in [4, 5]) to facilitate the readability.

#### I.I-3 Set Up Unit System

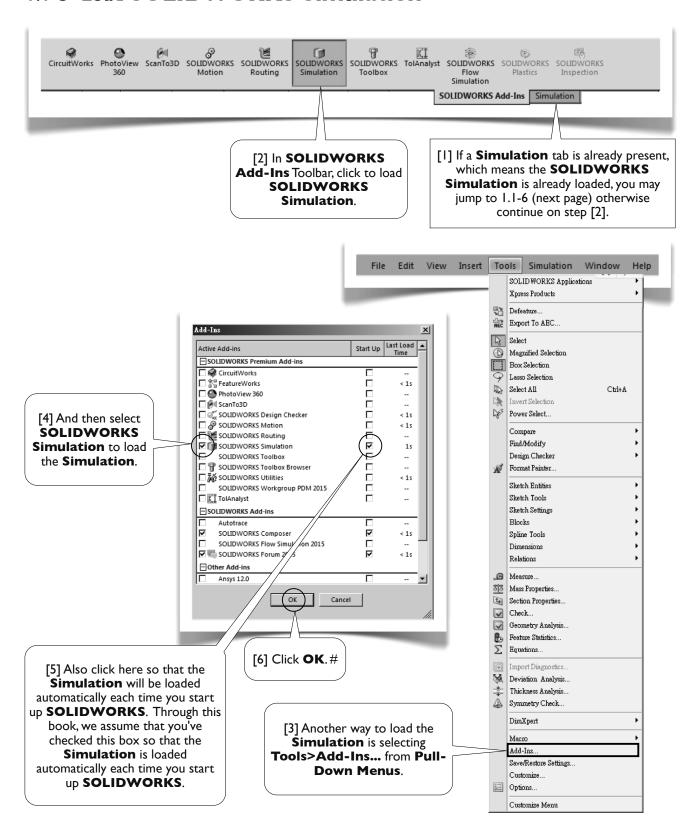


#### 1.1-4 Create Geometric Model

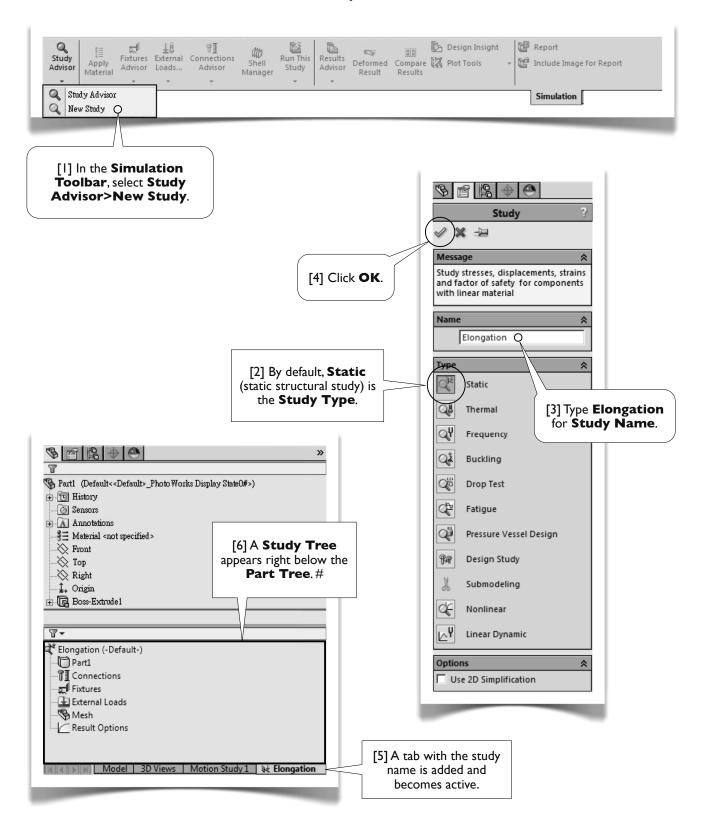




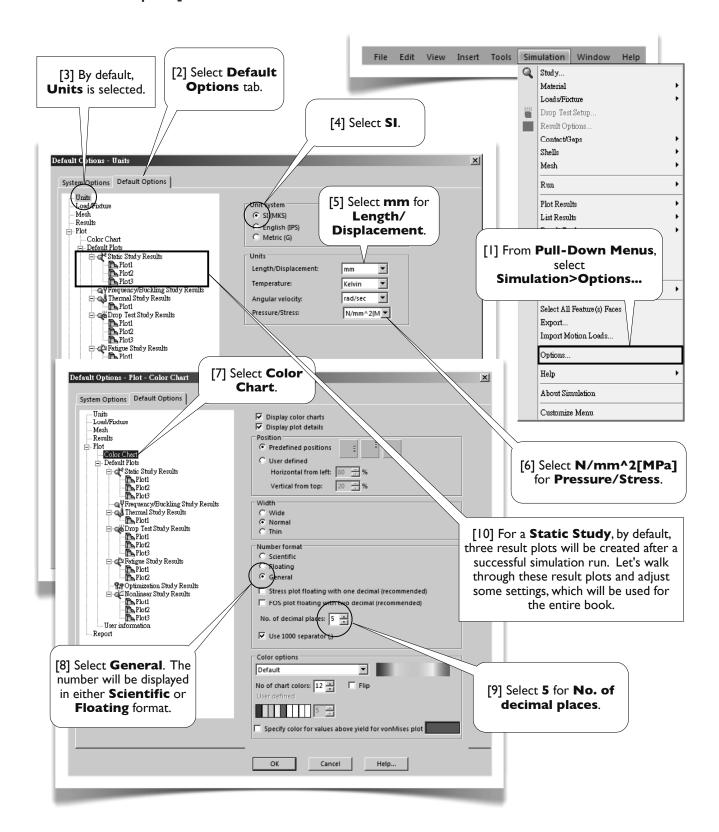
#### 1.1-5 Load **SOLIDWORKS Simulation**

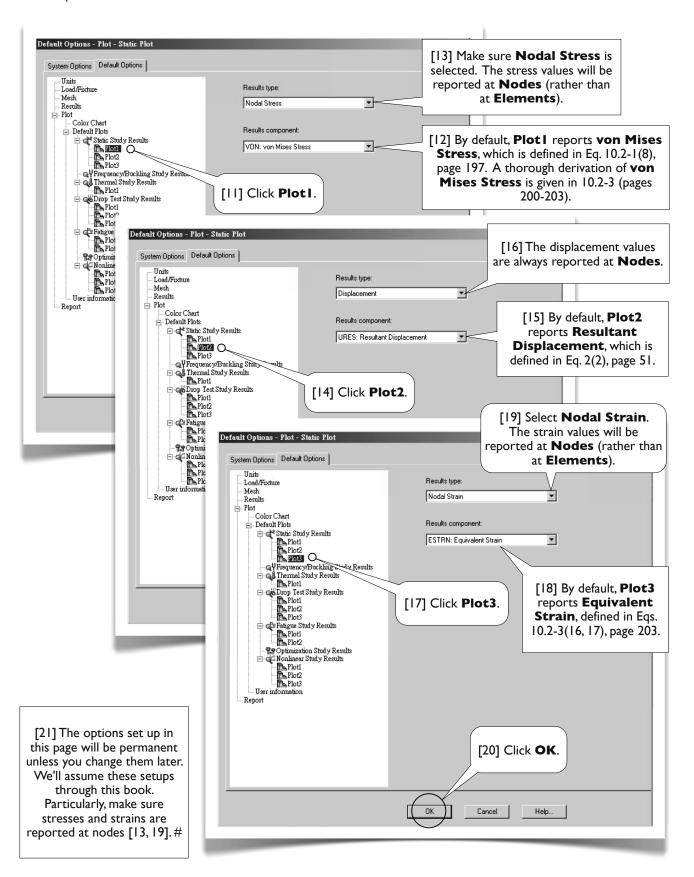


#### 1.1-6 Create a Static Structural Study



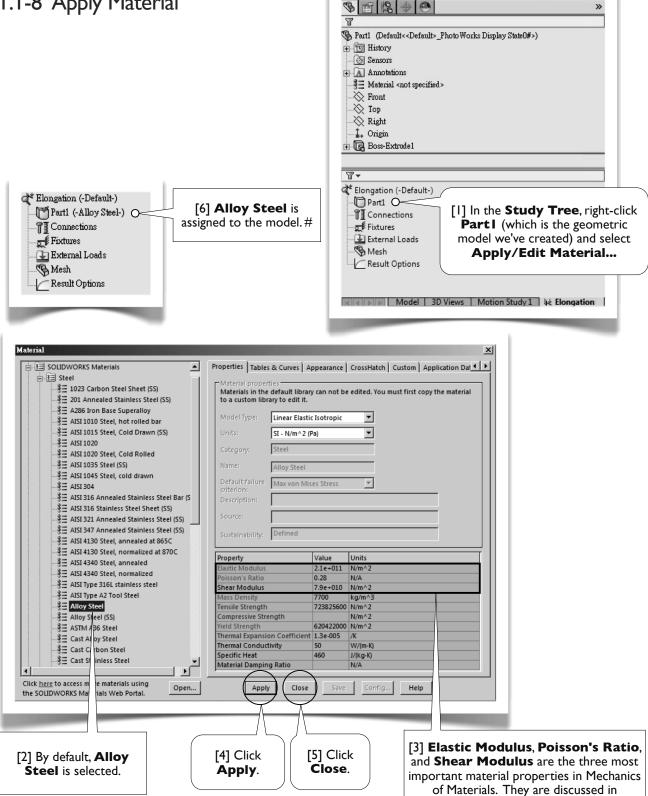
#### 1.1-7 Set Up Options for SOLIDWORKS Simulation



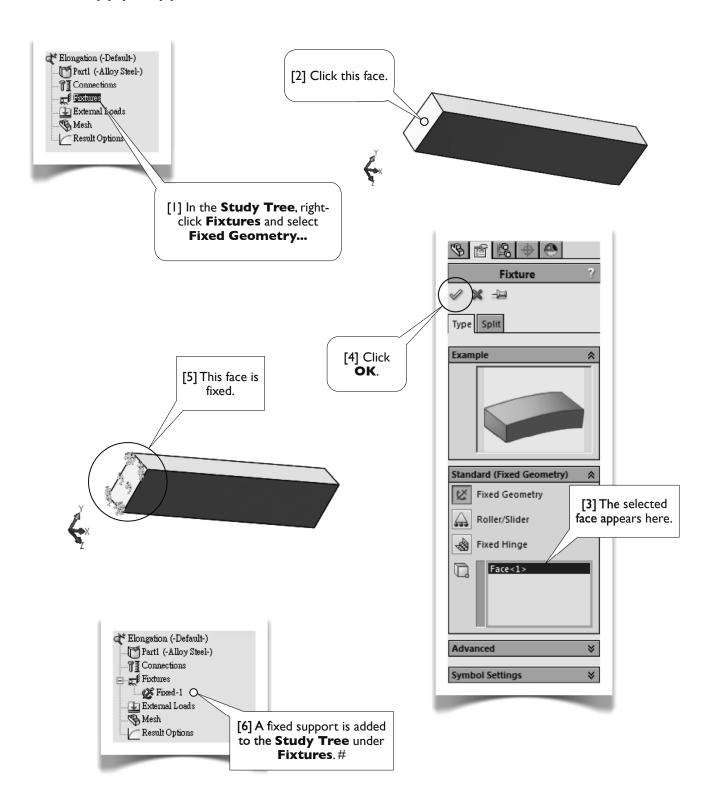


Sections 4.1 and 4.2.

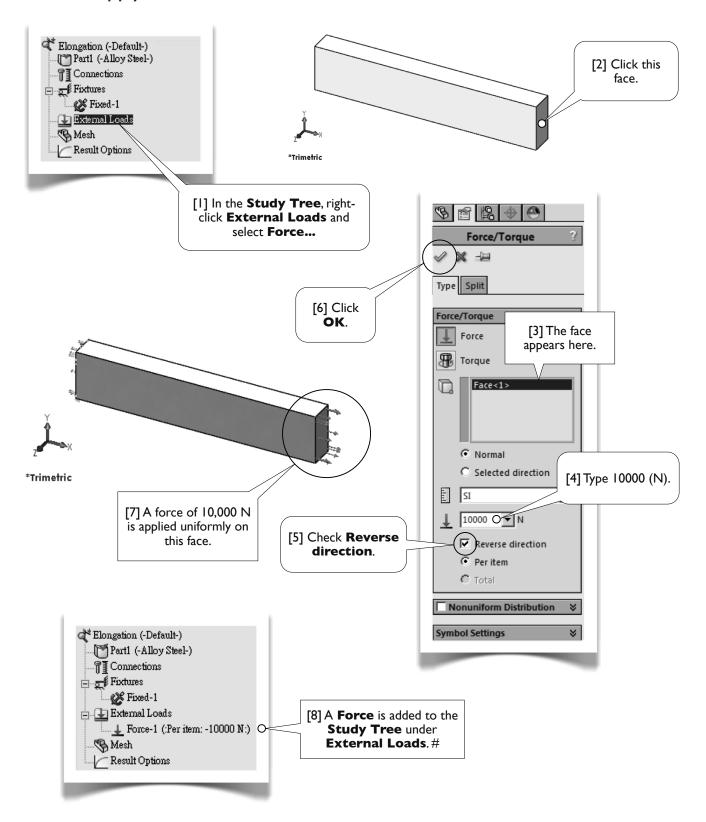
#### I.I-8 Apply Material

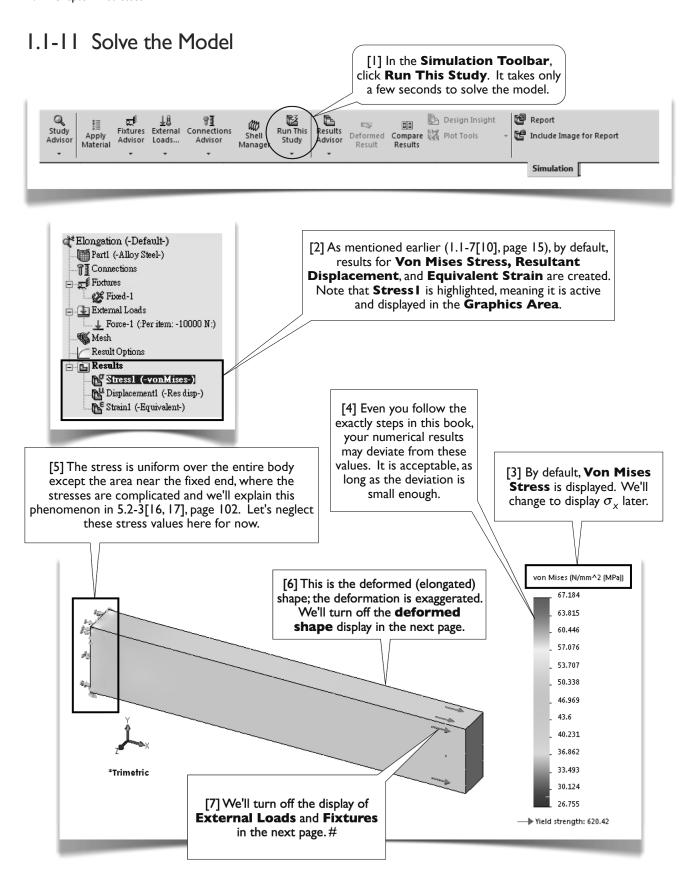


#### I.I-9 Apply Support

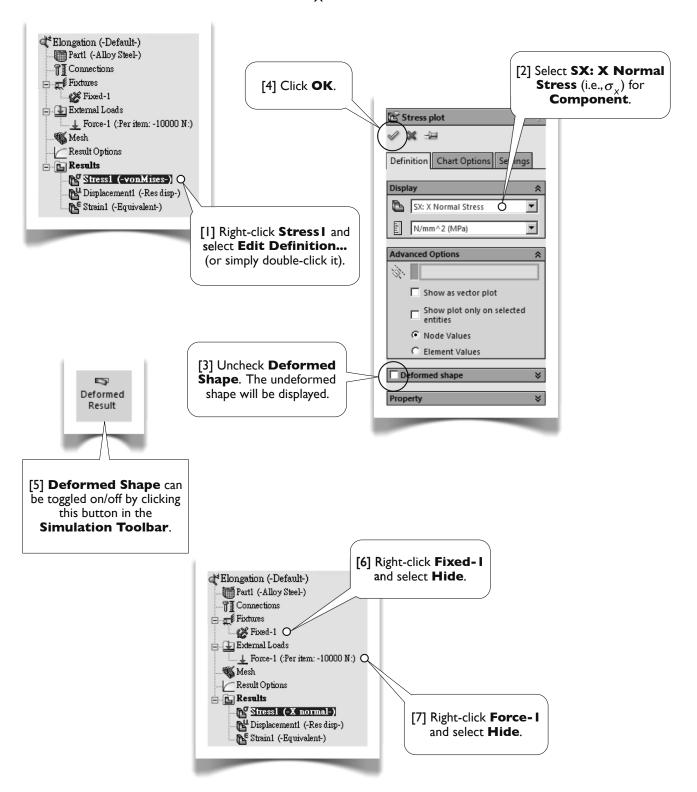


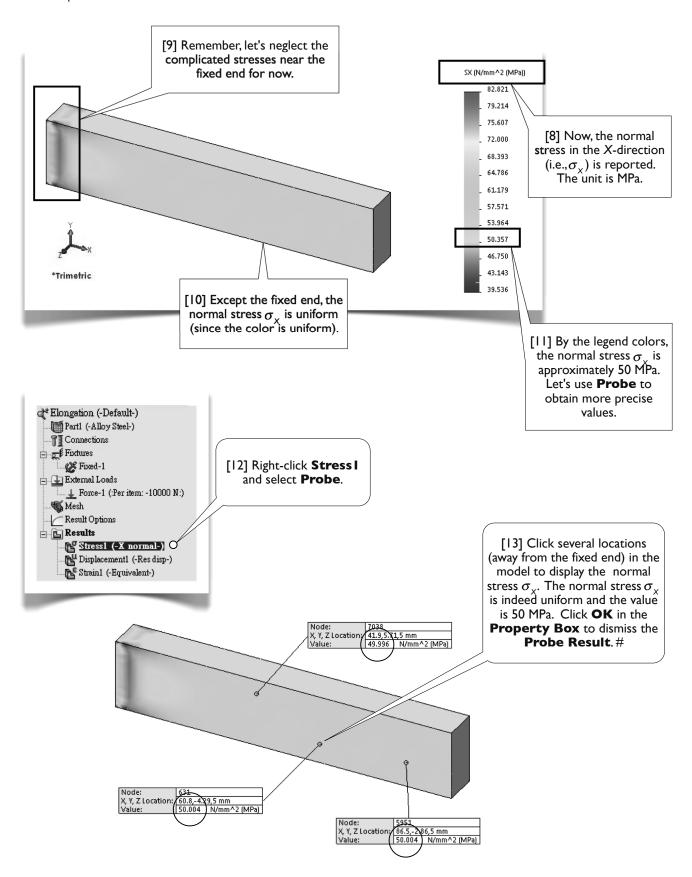
#### 1.1-10 Apply Load



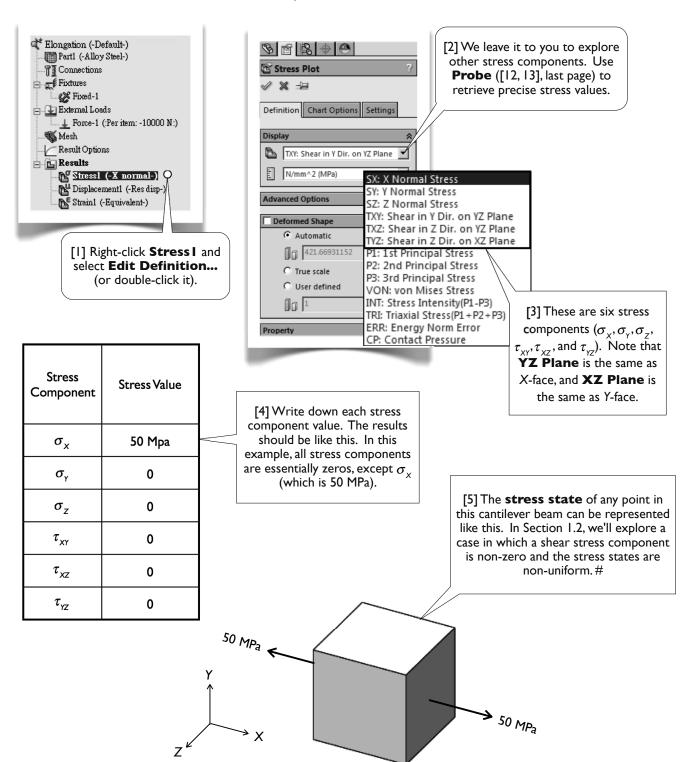


#### 1.1-12 View the Normal Stress $\sigma_{_{ imes}}$

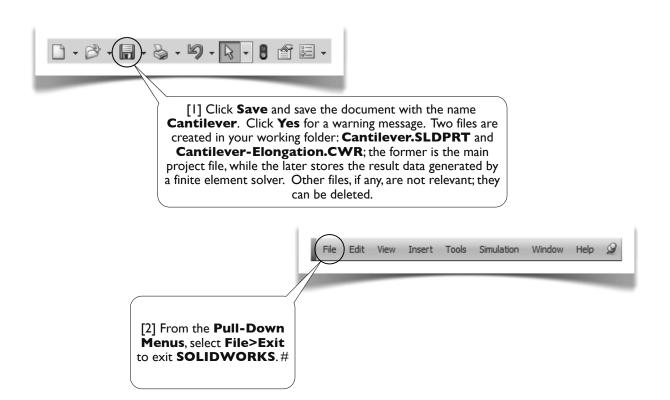




#### 1.1-13 View Other Stress Components

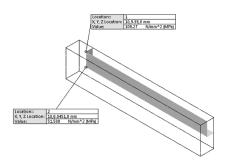


#### 1.1-14 Save the Document and Exit SOLIDWORKS



# Section 1.2

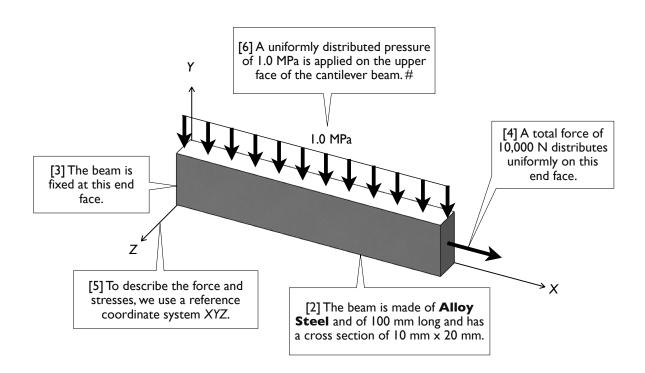
#### Nonuniform Stresses



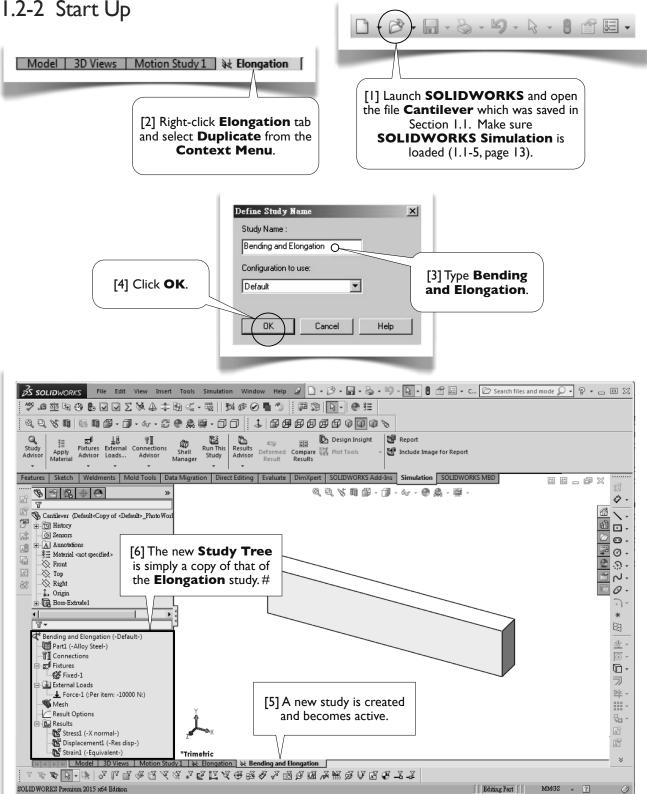
#### 1.2-1 Introduction

[1] In the last section, the stress field was uniform over the body and the only non-zero stress component was  $\sigma_{\chi}$ . In this section, we'll use the same model as in the last section [2-5], but add a uniformly distributed transversal pressure of 1.0 MPa on the upper face of the beam [6]. In this case, non-zero shear stress components exist in the beam and the stress field will not be uniform any more.

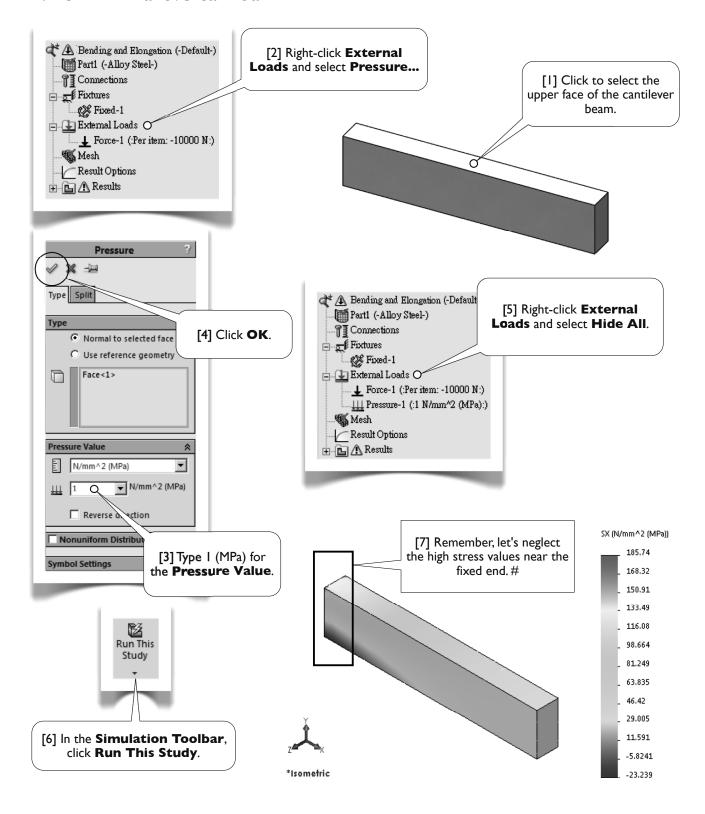
This section also demonstrates a way to retrieve results at specific locations in a body, namely the **Section Clipping** method.



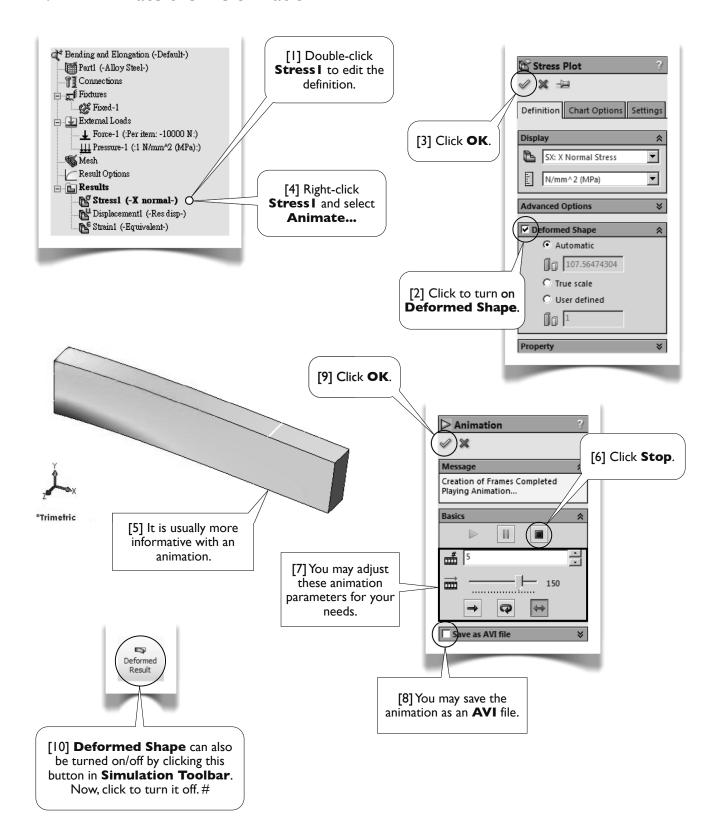
#### 1.2-2 Start Up



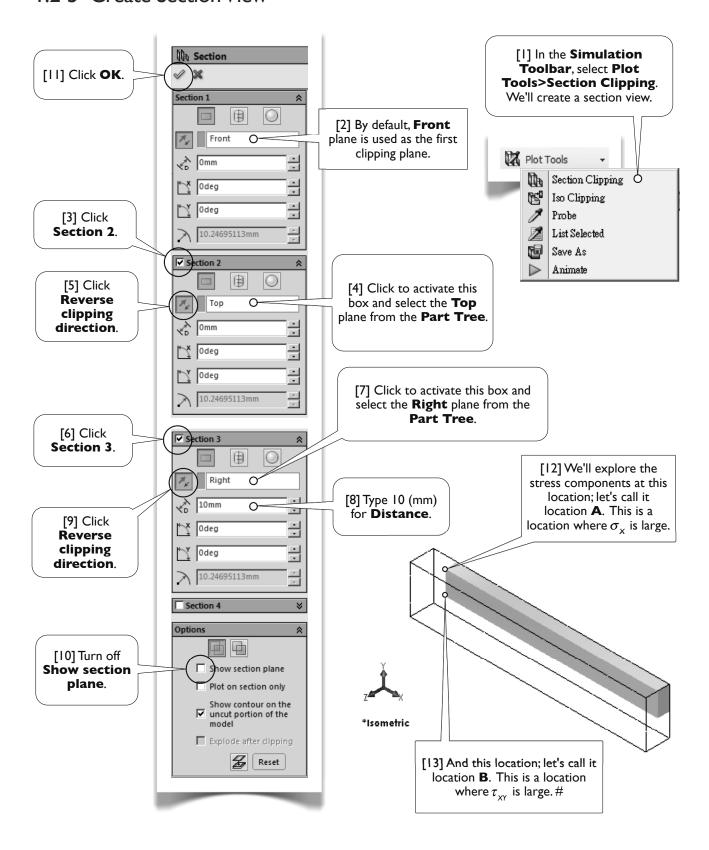
#### 1.2-3 Add Transversal Load



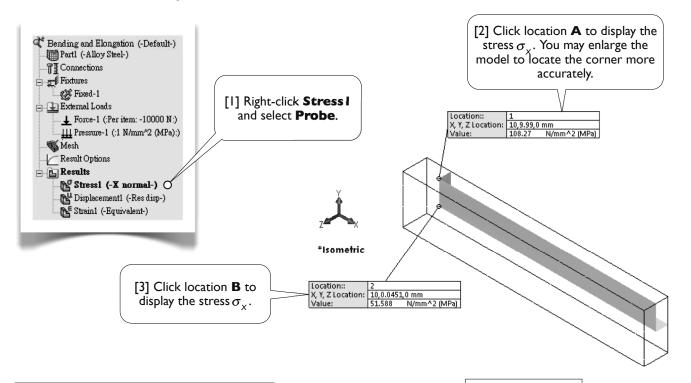
#### 1.2-4 Animate the Deformation



#### 1.2-5 Create Section View

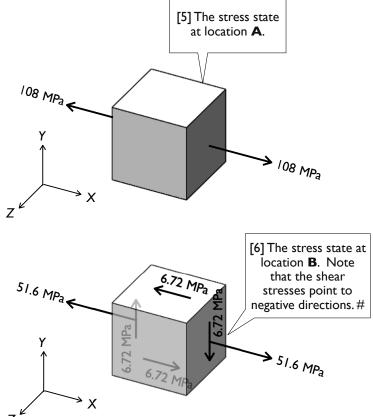


#### 1.2-6 Stress Components at the Locations A and B

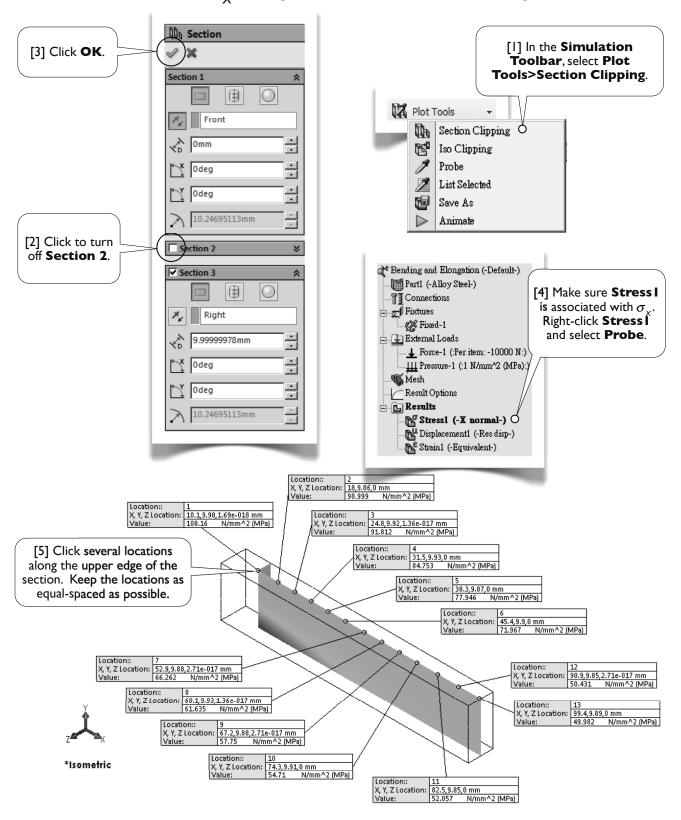


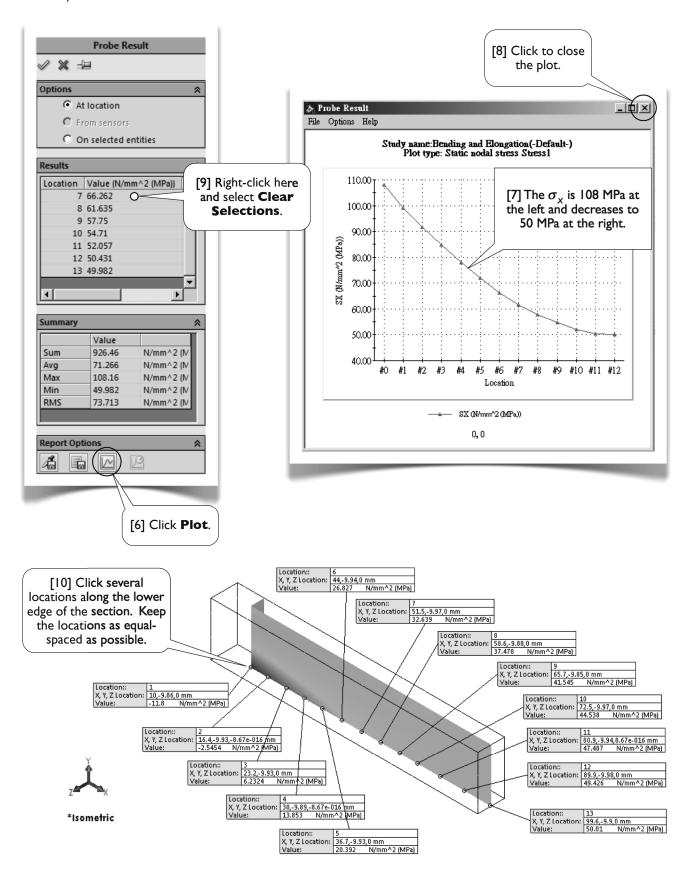
Stress Component	Location <b>A</b>	Location <b>B</b>
$\sigma_{_{X}}$	108.27 Mpa	51.588 Mpa
$\sigma_{_{_{ m Y}}}$	0	0
$\sigma_z$	0	0
$ au_{XY}$	0	-6.724 MPa
$ au_{XZ}$	0	0
$ au_{_{Y\!Z}}$	0	0

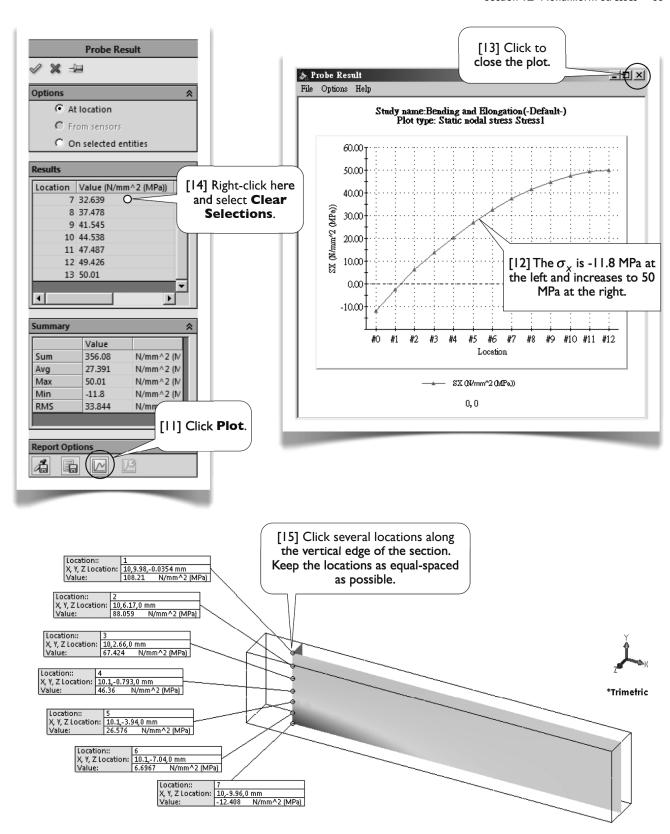
[4] Use **Probe** to explore other stress components and tabulate the data like this. Your stress values may not be exactly the same as here. Note that the shear stress  $\tau_{\rm XY}$  at location **B** is negative.

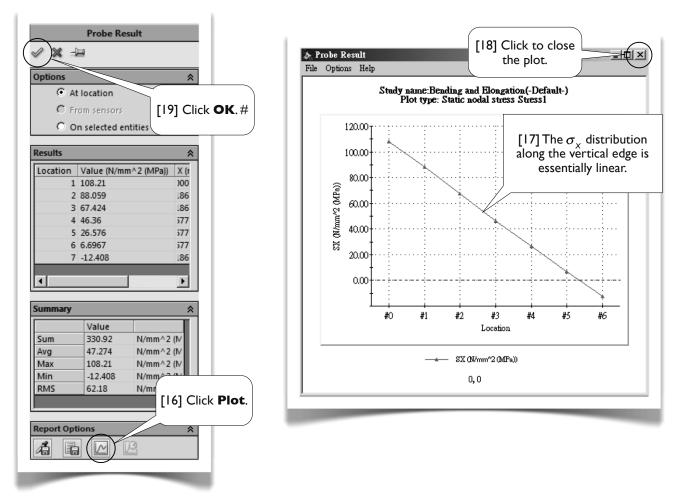


#### 1.2-7 Distribution of $\sigma_x$ Along Horizontal and Vertical Edges

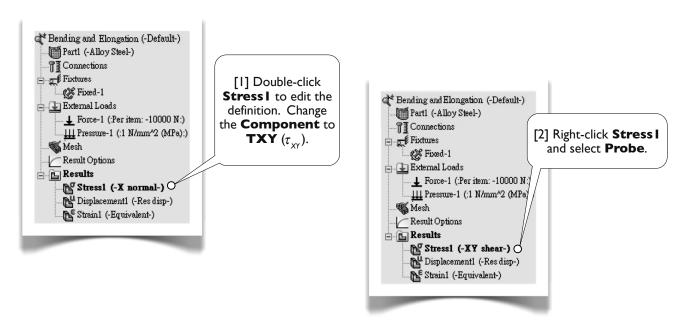


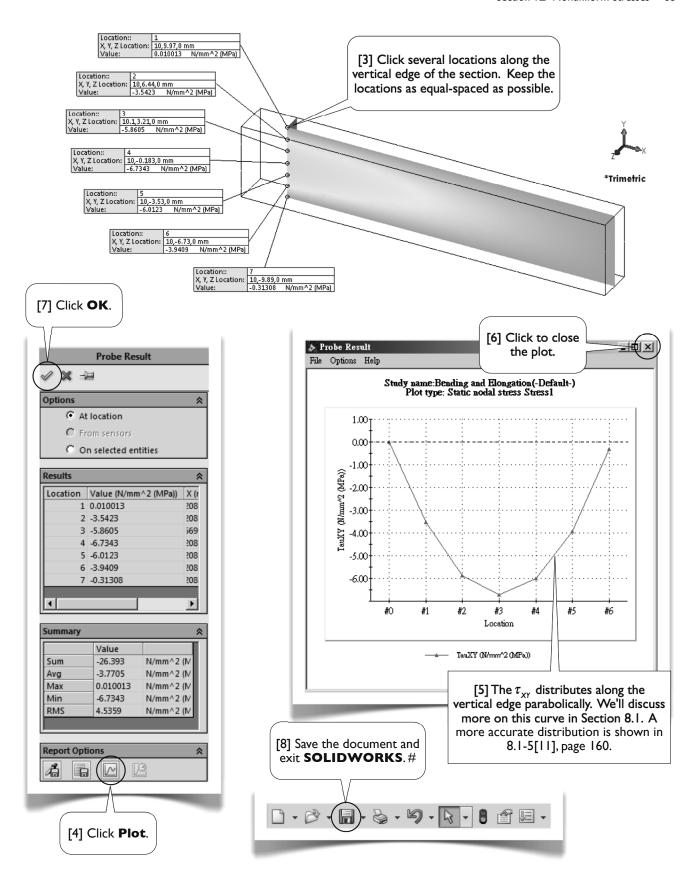






#### 1.2-8 Distribution of $au_{\rm XY}$ Along a Vertical Edge

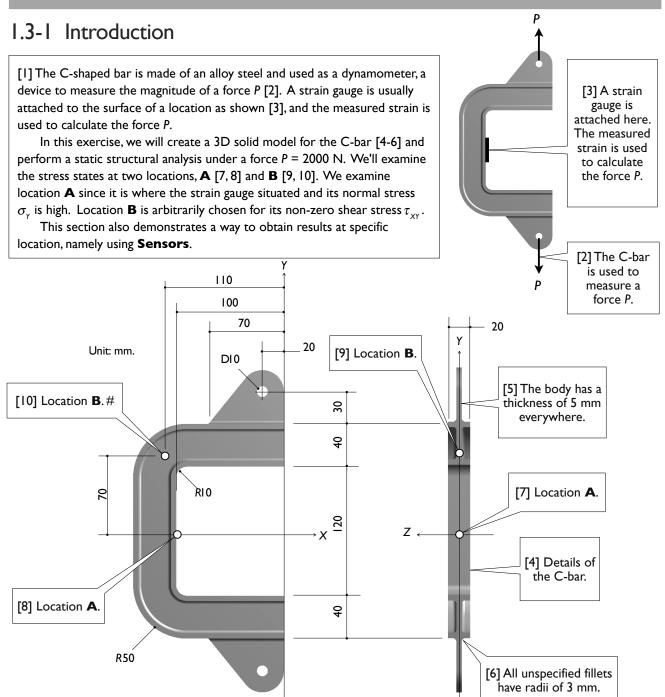




# Section 1.3

#### Stresses in a C-Bar

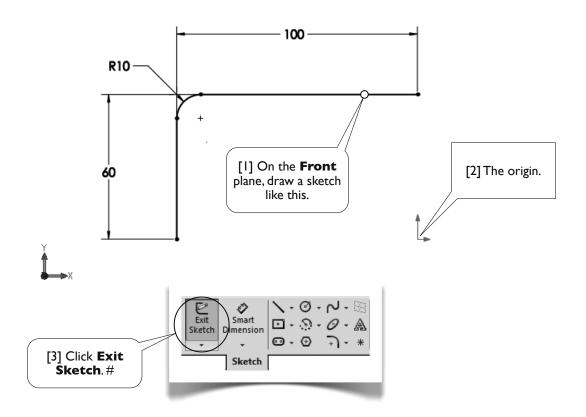


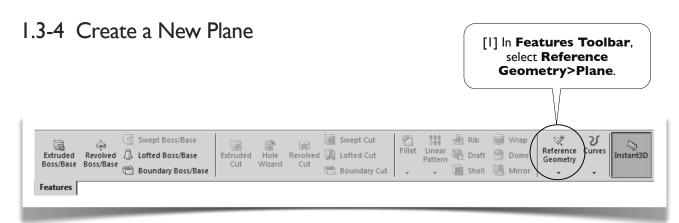


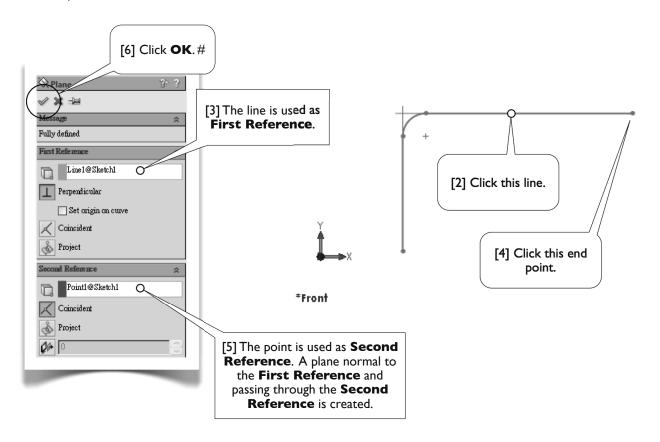
#### 1.3-2 Start Up

[1] Launch **SOLIDWORKS** and create a new part. Set up **MMGS** unit system with zero decimal places for the length unit.#

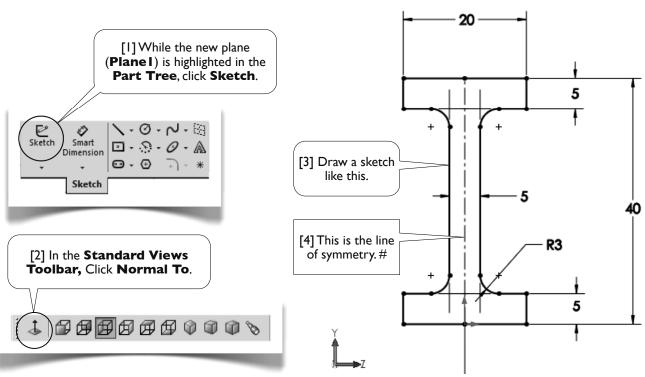
#### 1.3-3 Create a Sketch for the Sweeping Path



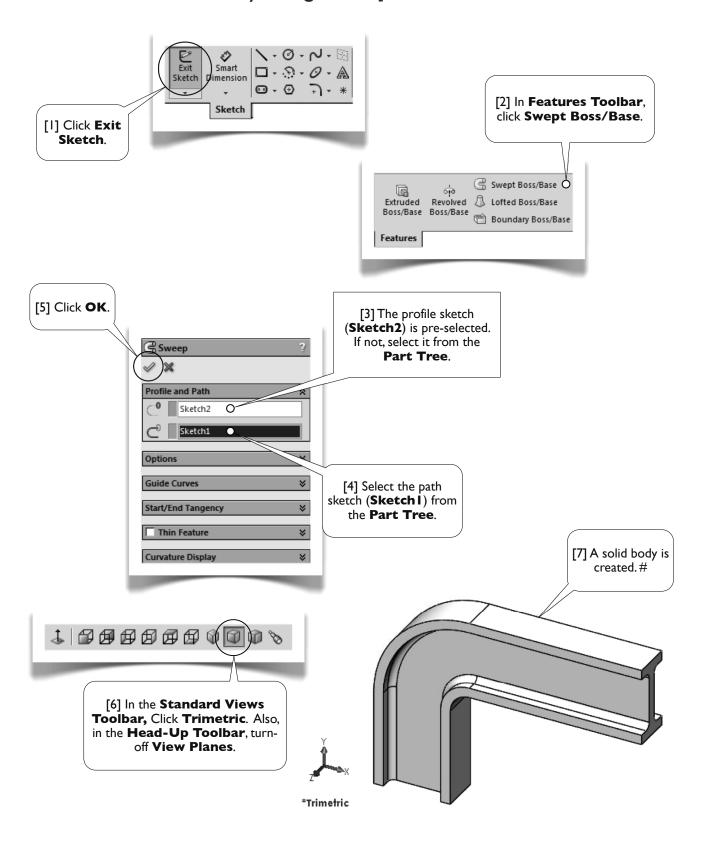




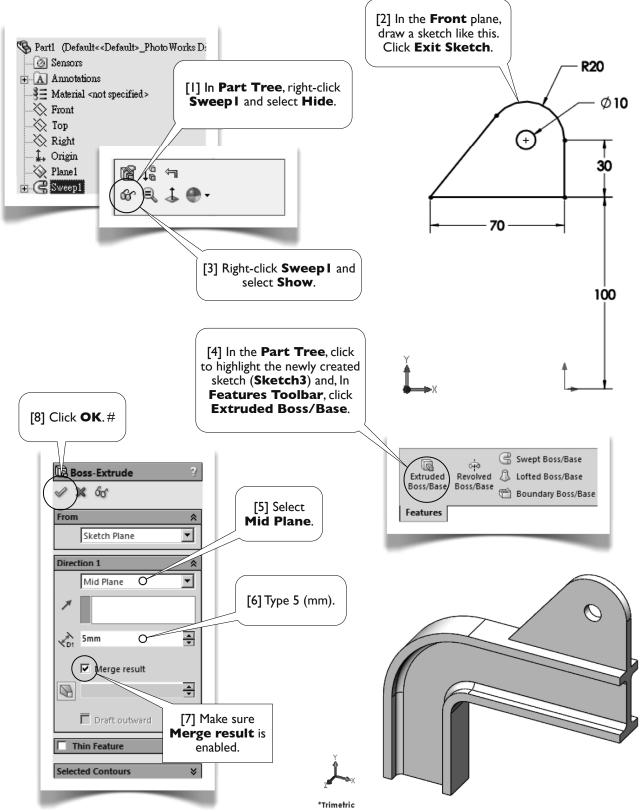
#### 1.3-5 Create a Sketch for the Profile



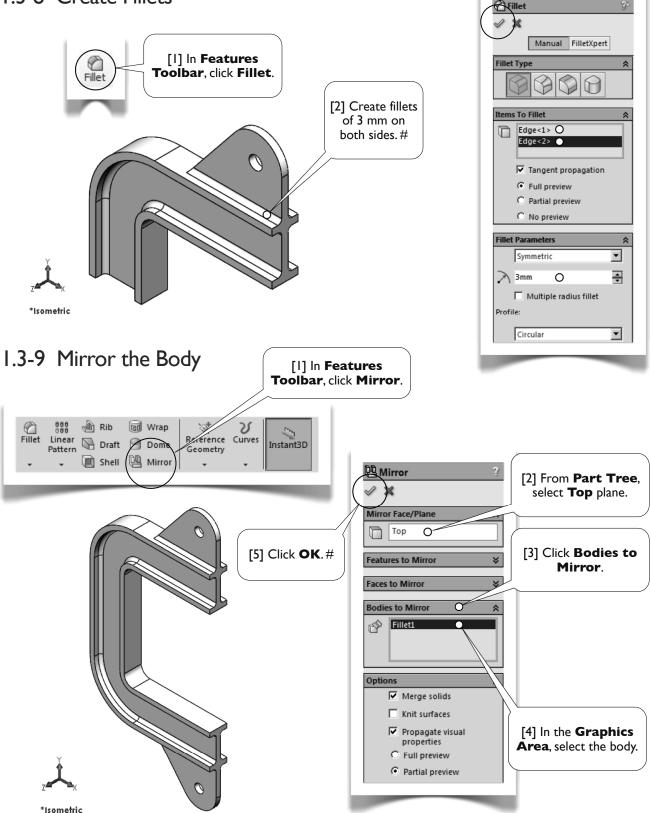
# 1.3-6 Create a Solid Body Using Sweep

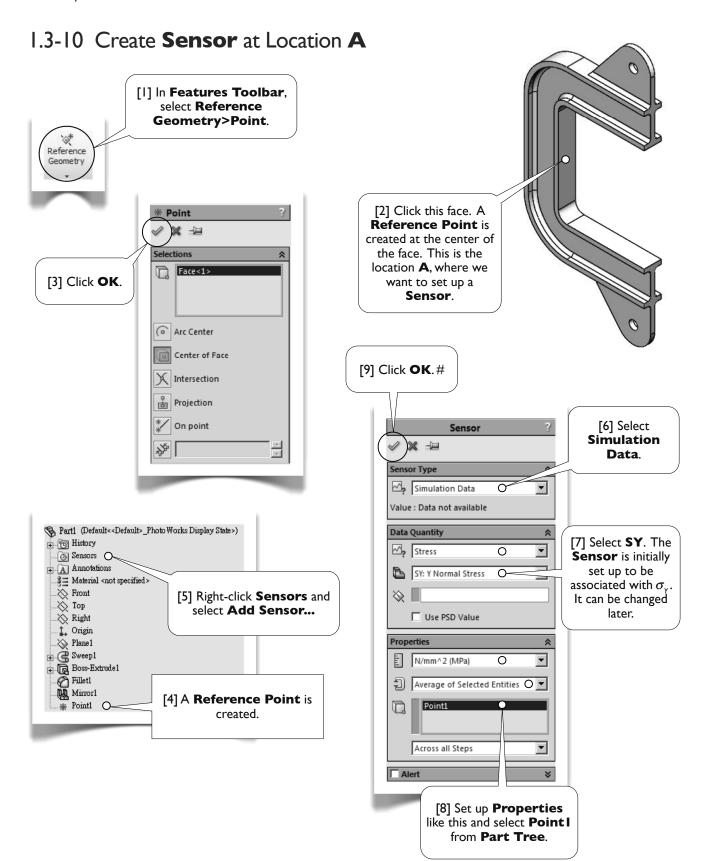




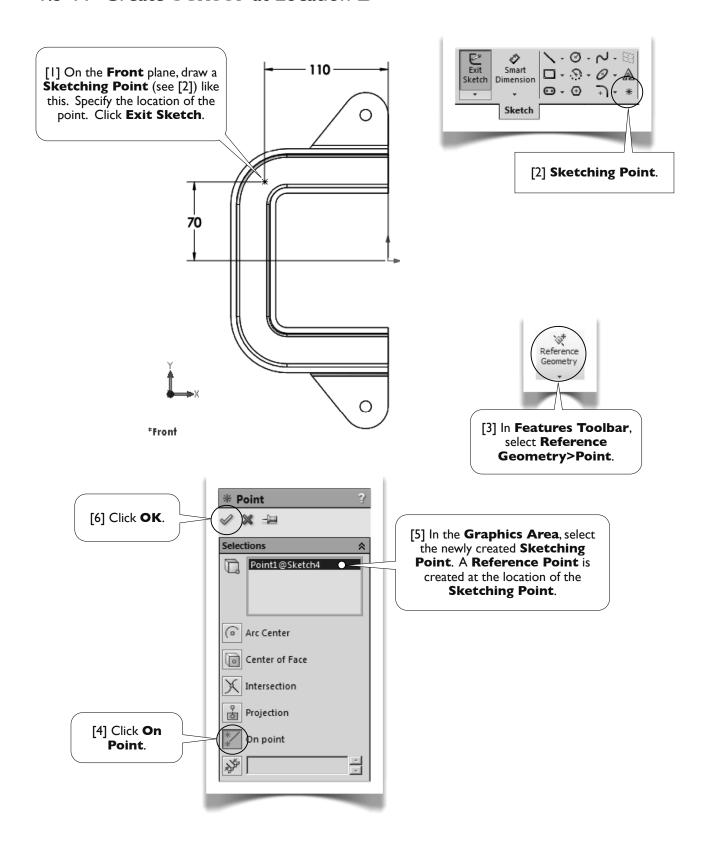


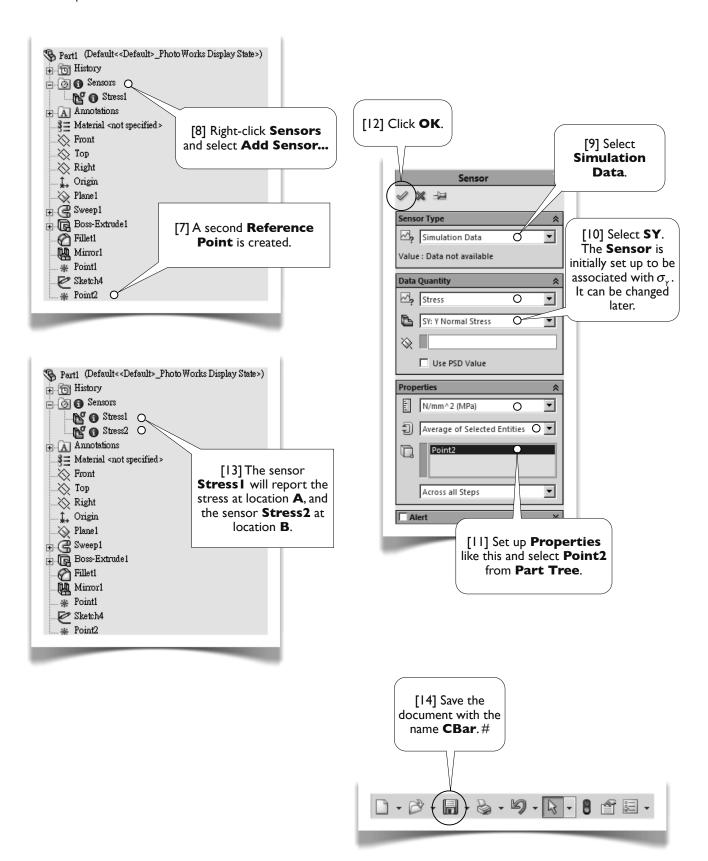
#### 1.3-8 Create Fillets



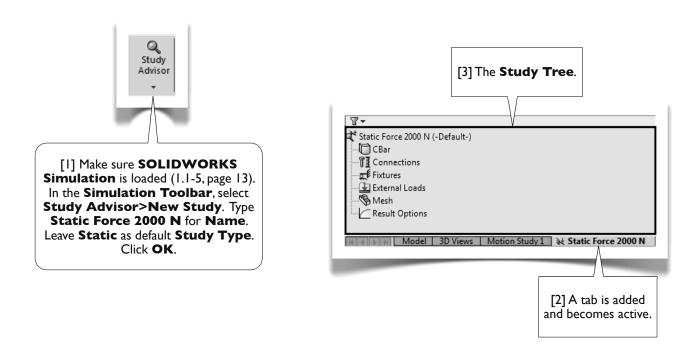


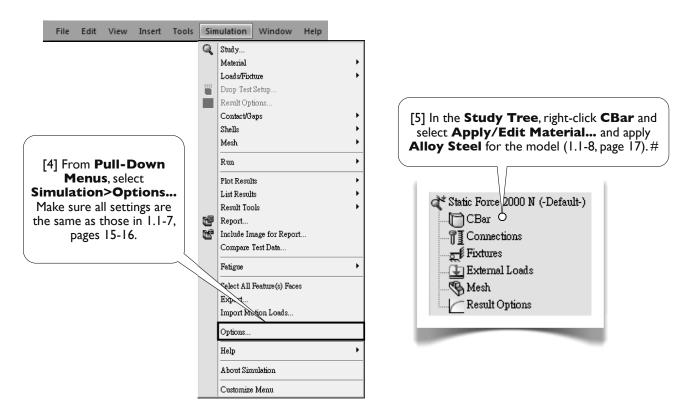
#### 1.3-11 Create **Sensor** at Location **B**

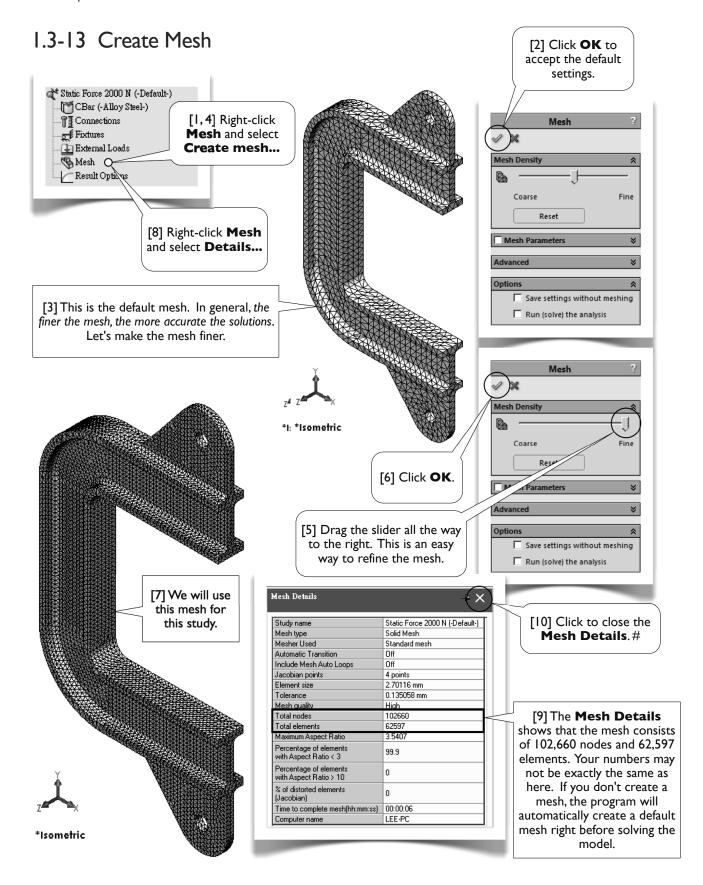




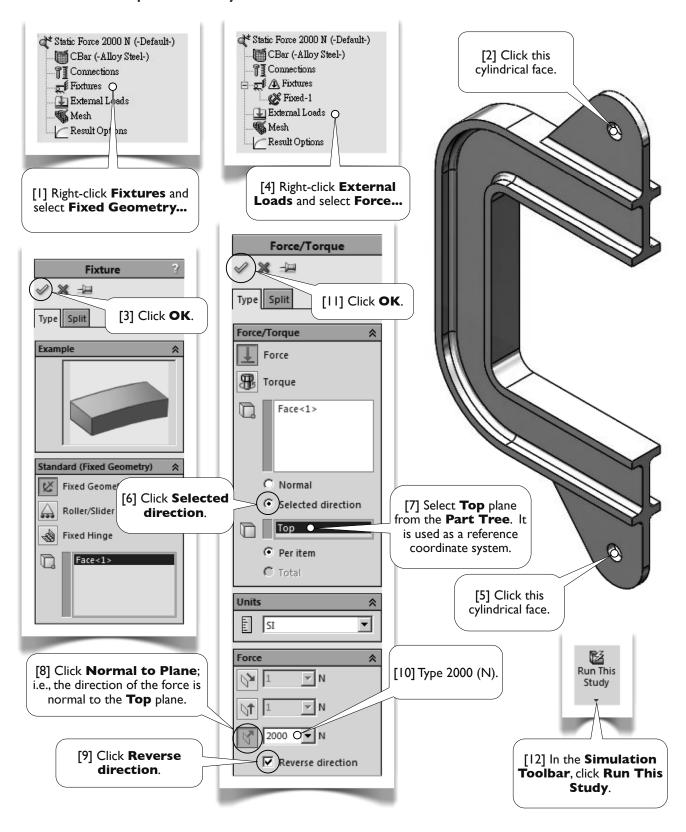
## 1.3-12 Create a Static Structural Study and Set Up Unit System

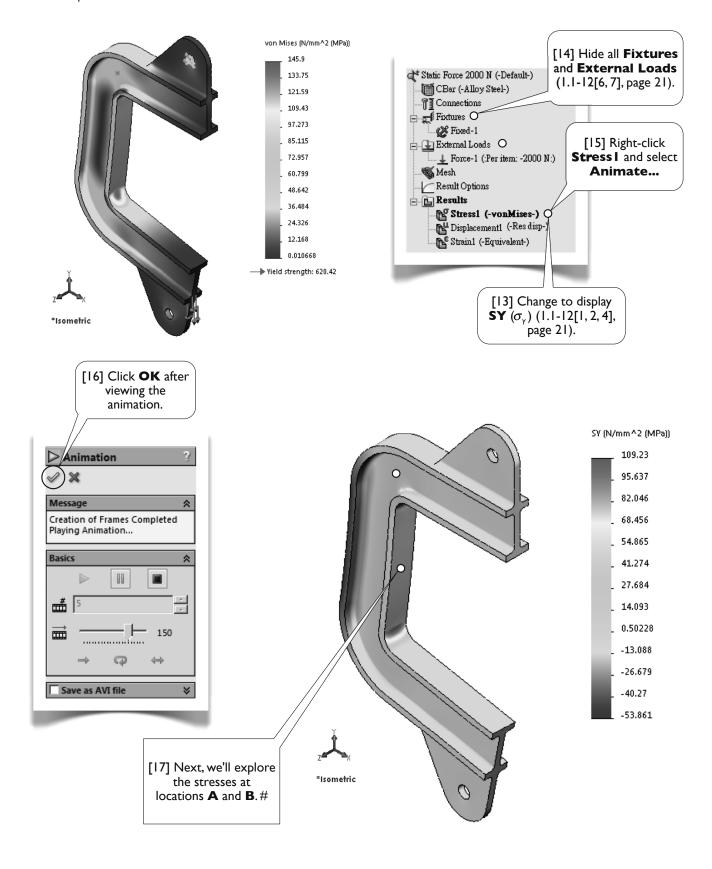




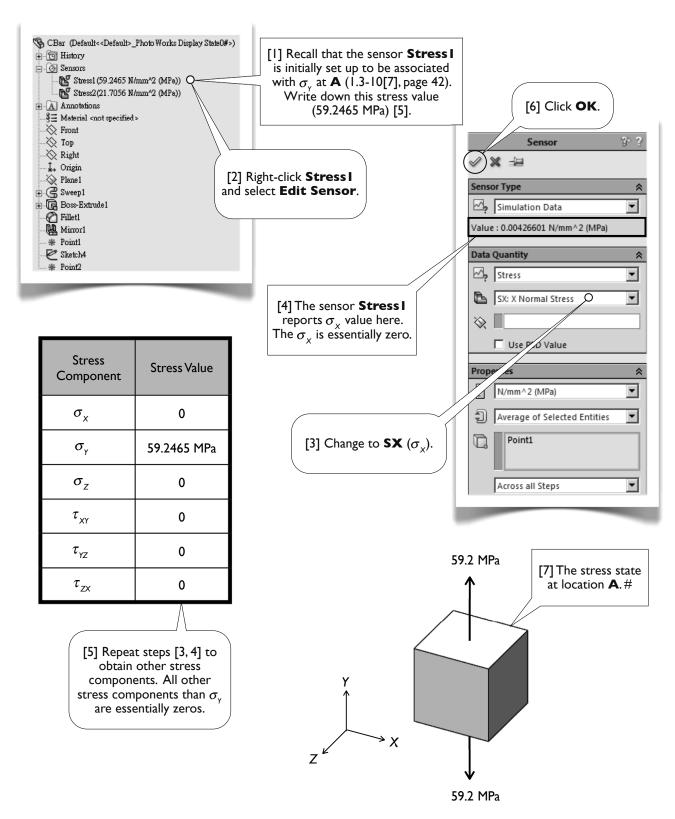


## 1.3-14 Set Up Boundary Conditions and Run the Model





### 1.3-15 The Stresses at Location A



### 1.3-16 The Stresses at Location **B**

